Spacetimes with and without Universal Connections

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Abstract

Everyone can agree to the fact that the observable spacetime can be formally represented by a four dimensional differential manifold, globally oriented and, for the moment, structureless. But to describe physical phenomena is, at least, necessary to assign to our manifold a ***linear connection*** and, then, a covariant derivative, a notion of parallel transport and of geodesic, a torsion and a curvature tensor. However, at this point, we are confronted to two different approaches, methodologically quite incompatible.

In the first one (Poincaré, Einstein 1905, Minkowsky) we assume that the spacetime manifold is endowed with a globally flat, torsion-less ***universal*** (background) connection: tangent spaces at different spacetime point can now be identified with a unique, four dimensional vector space; on this vector space we have a quadratic form, non-degenerate, symmetric and of signature ±2; the vector space is also time oriented. So the affine space that we finally obtain is a Minkowsky spacetime; the geodesics are straight lines (unidimensional affine submanifolds) and the time-like ones describe the behaviour of (ideal) non interacting particles, in absence of gravity. We can now construct relativistic Quantum Theories (Quantum Field Theories) and explain, more or less, every physical fact, obviously excluding gravitation.

In the second approach (Hilbert, Einstein 1915) the spacetime connection is simply a variable physical quantity; this connection depends on a (classical, pre-quantal) matter energy-momentum tensor and, generally, it is not flat; this manifold is also time oriented and the time-like geodesics describe the behaviour of (ideal) non interacting particles, in presence of gravity. So we shall call this connection ***gravitational***, opposed to universal. Apparently, it is now not possible to construct some relativistic Quantum Theory; we can theoretically explain, strictly speaking, only gravitation.

While, for a conceptual point of view, the situation is quite embarrassing, practically there is no problem. In fact we can build (mainly quantum) objects, considering also, in some phenomenological way, gravitational effect; a nice example is the GPS system.

But we have to examine the problem from the experimental side, the only one methodologically (and philosophical) relevant. First at all, we need some quantum mechanical clock, which is a quantum device with a characteristic time (or frequency). As a peculiar type of clock we can choose a batch of instable, decaying particles: if we transport the batch along a world line, a time-like curve, the ratio between the average number of particles at the starting (spacetime) point and the same number at the end (spacetime) point depend on the path. More exactly, the logarithm of the above ratio is proportional to some path length. We remember that the particles number is an "invariant", we don't need any reference frame. Typical curves are the time-like helices (corresponding to spatial circular orbit, as in particles accelerators or satellites and planets), broken time-like lines (as in a spatial return journey) and so on.

To complete our experiment, we transport identical batch of particles (the "twins") along different time-like curve connecting the same pair of spacetime points: there is one and only one most decayed batch (the oldest twin); the corresponding path is our geodesic, the related connection can be or not to be flat. And the above mentioned path length (proportional to the logarithm of the number of particles ratio) has to be calculated using the gravitational, contest dependent, connection; in fact using the related pseudo-metric symmetric tensor.

Can we now necessarily conclude that we have to describe the physical world employing only the gravitational, variable connection? We believe, absolutely not. We can safely equip our spacetime with ***two connections***: a globally flat one, universal and a variable one, locally curved. The Quantum Theories has to be formulated considering also both connections; quantum effects, as the timing of quantum clocks, should be gravity dependent, as required. Finally we observe that, because a difference of two connections is always associable to a field (a (1,2)-type field, in fact), we can describe the spacetime as an affine, Minkowsky space plus a gravitational field.