On the Ontology of the Quantum Structure of Space-Time

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Abstract

We discuss the mathematical and philosophical origin of quantum structure of space-time. We interpret the expansion of the space (universe) as the expansion of the possibilities (in the probability space). We also look into the statistical origin of distance and metricity and an extended framework of the generalized quantum mechanics in search of quantum structure of space-time. The idea of distinguishability was proposed by Wooters to define the notion of distance, called "statistical distance". The definition involves counting the number of distinguishable states between two given quantum states, when all states are analyzed by the same measuring device. Statistical distance is determined entirely by the size of statistical fluctuations, and has nothing to do with the usual distance between two pure states, that is, the angle between rays in a Hilbert space. However, it is coincidence that nature mysteriously makes these two notions of distance identical. This distance between two statistical events is given in terms of the number of distinguishable events, thus forming the space with the associated Riemannian metric. Having known the fact already discovered that the statistical distance in the probability space and the notion of distance in the ray space on $CP(N)$ are identical up to a multiplicative constant $\hbar$, we have yet another interpretation which will have far reaching consequences. The notion of distinguishability valid for the metric of quantum states in the ray space on $CP(N)$ is valid even for the metric of the quantum states on the Reimannian space (manifold) with real co-ordinates x^{μ}. We argue that dimensionless quantity ds^2 and the statistical distance in the probability space ought to be identical up to a multiplicative constant $\lambda\!\!\!^{-}_C$ - that is Compton's Wavelength. And thus we suggest the context may be further examined for the possible co-relation of this equivalence with the metric in the configuration space of

space-time (product space), whenever the quantum state space (real) coincides with the configuration space.

And thus one gets the feeling of the space-time spanning with the discreteness of the order of Compton's wavelength (or Planck's length, as the lowest limit the Compton's wavelength ceases to be is the Planck's length). It makes physical sense too. We interpret the expansion of the space (space-time) as the expansion of the possibilities (in the probability space).

We state the fundamental prerequisites of the construction of a dynamical space, and thus we suggest prescription of a Posteriori which answers the very basic questions.

Priories: (i) Separability should be well defined.

(ii) Differentiability, and hence continuity must be admissible.

(iii) Triangle inequality: $s(p, p') + s(p', p'') \geq s(p, p'')$, holds; and

(iv) Isometry should be satisfied.

Posteriori: The explanation of the theoretical construction of the space (space-time)- how the space expands? And, where does it expand?

The proposition in this paper precisely pertains to these questions.

Mathematically, the space-time spanning indefinitely is digestible. But, the real challenge lies in justifying the physical expansion of the universe. The expansion of the space with the possibilities (in the probability space), though logical, sounds rather philosophical. However, from the quantum mechanics point of view, the expansion of the probability space makes genuine physical sense. Like space-time, the causality has to be an essential ingredient in the spanning of the probability space too.