

Non-locality and Stochasticity on the Time Axes

Toru Ohira

Sony Computer Science Laboratories, Inc.

Tokyo, 141-0022, Japan

ohira@csl.sony.co.jp

<http://www.sonycsl.co.jp/person/ohira.html>

The main theme of this paper is to consider concepts of “stochasticity” and “non-locality” on the time axes within the framework of classical dynamics through a presentation of rather simple models. These concepts are normally associated in the “space” variable, not in the time variable. We try to illustrate here the question of what happens if we transfer these concepts on the time axes.

More concretely, time is normally viewed as not having stochastic characteristics. This is so in normal dynamical systems, whether they are classical, quantum, or relativistic. In stochastic dynamical theories, we consider noise and fluctuations with only “space” variables, such as the position of a particle, and not with the time variable. In quantum mechanics, the concept of fluctuation is embodied in the time-energy uncertainty principle. However, time is not a dynamical quantum observable, and clear understanding of the time-energy uncertainty has yet to be found.

The similar situations in our cognition of non-locality in space and time. Non-local effects in space are incorporated in physical theories through wave propagation, fields, and so on. In quantum mechanics, the issue of spatial non-locality is more subtle, as appearing in the Einstein-Podolsky-Rosen paradox. With respect to time, there have been investigations of memory or

delay effects in dynamical equations. In general, however, less attention has been paid to non-locality in time, and behaviors associated with non-locality in time, such as delay differential equations, are not yet fully understood.

Against this background, we present simple classical dynamical models to illustrate the idea of introducing a stochasticity with non-locality into the time variable. For non-locality in time, we discuss delayed and anticipating dynamics which involve two points separated on the time axis. Similarly with respect to stochasticity, we discuss a model which includes noise in the time variable but not in the space variable. With certain combinations of fluctuations and non-locality on the time axis, we observe a “resonance” effect. This effect is similar to “stochastic resonance”, which arises through a combination of oscillating behavior and spatial noise and has been studied in variety of fields. We would like to discuss how these models may be developed to fit a broader context of generalized dynamical systems where fluctuations and non-locality are present in both space and time.

References:

"Stochasticity and Non-locality of Time", T. Ohira, *Physica A*, vol. 379, 483, (2007).

"Resonance with Noise and Delay", T. Ohira and Y. Sato, *Physical Review Letters*, vol. 82, 2811 (1999).

"Delayed Stochastic Systems", T. Ohira and T. Yamane, *Physical Review E*, vol. 61, 1247 (2000).