Weyl's refutation of conventionalism

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In *Space, Time, Matter (RZM)* Hermann Weyl presents his own contribution toward solving the "space problem" for spaces of 2 or 3 dimensions, and a speculative extension of that contribution into spaces of any integral dimension. He later establishes the mathematical basis for that extension, thus moving it out of the realm of the speculative, in "The Uniqueness of the Pythagorean Metric." As Weyl characterizes it in *RZM* the space problem is to explain why the metric of physical space (spacetime) is infinitesimally pythagorean. His solution is to show that requiring that very small displacements and rotations of bodies constitute a mathematical group of operations. His demonstration that only in infinitesimally pythagorean spaces can this requirement be met is essentially an infinitesimal version of the Helmholtz-Lie theorem establishing that free-mobility of bodies is possible only in spaces of constant curvature.

Weyl however takes himself to have established much more than that. He argues that he has shown that only in spacetimes that admit virtual changes in the metric can we demand not only that the metric be locally pythagorean, but the even more basic demands that it involves no cross terms, and no terms of higher order than quadratic. Here I will present Weyl's argument, assess it favorably, and then explain how it may be used to refute Carnap's early position on the conventional character of spatial geometry. That conventionalism is quite minimalist however, and so conventionalism in general appears implausible on this analysis.

I first introduce the space problem, as understood by Weyl and his contemporaries (and their immediate predecessors). I next present Carnap's solution to the problem, and show his conclusion that there are no metric facts in physical space to be proof against a class of responses that amount to inference to the best explanation—this class seems to be the most plausible response that has been offered to date. I then outline Weyl's response, and reconstruct on his behalf an argument that his solution demonstrates the factual character of the metric of space. By analogy with "inference to the best explanation" I call the form of argument "inference to the implicit explanans". I will try to make it appear plausible that such an inference is generally available as a response to conventionalist challenges.