Rigidity in Relativity

Stephen N. Lyle

The aim here is to discuss the problem of rigidity in relativity from two different angles: in the first part (Sect. 1), it is discussed as a property of a physical body in special relativity theory, as often assumed for calculation of self-force when the body is electrically charged; in the second (Sect. 2), it is approached as a property of the motion of a continuous medium, in a way that can be extended to general relativity theory.

The first considers the problem of a rod accelerating along its axis (Sect. 1.2), proposing a simple differential equation that might be expected to encapsulate the notion of rigidity (Sect. 1.3). The equation is solved by considering the standard (Rindler) coordinate frame adapted to the motion of an observer accelerating in a straight line: indeed it is noted that the distance between any two points with constant spatial coordinates for the accelerating observer in this frame, when reckoned in an inertial frame, satisfies the proposed rigidity equation (Sects. 1.4 and 1.5). This suggests that, if the accelerating observer carried such a rod, it would always lie between the same two spatial coordinates of the accelerating frame. In this context, it is useful to picture the rod as a four-dimensional object in order to understand the views that might be adopted by different observers.

This leads to a discussion of some remarkable properties of these accelerating frames (Sect. 1.6), with reference to the specific case of a uniform acceleration to allow explicit calculation, and their consequences for the rigid rod (Sect. 1.7), i.e., one asks how such a rod must behave physically in order to remain rigid. In particular, it is asked whether there must be any superluminal transmission of speed or acceleration along the rod (Sect. 1.8). It is concluded that this is not necessarily the case, but that rigid motion may not always be possible, however a rod is constituted.

The problem of how measuring sticks should behave is relevant to the notion of proper distance, often treated lightly in textbooks. Likewise for the link between coordinates and physical measurement, rarely discussed explicitly. These questions are raised in Sect. 1.8 in preparation for further discussion in the second part of the paper.

Finally, reference is made to Bell's idea of calculating the shape of an electron orbit in an accelerating atom (Sect. 1.9). A priori, there is no reason why a rod made up of a string of such Bell atoms should turn out to be rigid in the above sense. However, it is argued that there is likely to be some approximate equality and this suggests a way of understanding how a rod might behave rigidly (according to the definition here).

The second part of the paper moves away from the idea of rigidity as an attribute of a body to rigidity as an attribute of the motion of a continuous medium, as described by DeWitt in his 1970 Stanford lectures. A general description of the motion of such a medium is obtained by labelling particles in it (Sect. 2.1). This leads naturally to the notion of proper metric describing the proper separation of neighbouring particles as viewed in the instantaneous rest frame of either. A rigid motion is then one in which the proper metric remains constant (Sect. 2.2). It is shown that the rigid rod of Sect. 1 is undergoing rigid motion in this sense.

More detailed calculations with general coordinates and the above-mentioned label coordinates are illustrated by introducing the rate of strain tensor (Sect. 2.3), which can also be used to characterise rigid motion: one has such a motion if and only if this tensor is zero. It is shown how to extend the ideas to general relativity theory.

Section 2.4 introduces a whole class of examples of rigid motions, showing just how restrictive this notion is in relativity theory. Section 2.5 discusses rigid motion when there is no rotation, showing the relevance of the idea of Fermi-Walker transport. Such motions have only three degrees of freedom. It is shown that the label coordinates constitute precisely the accelerating coordinates derived in Sect. 1.

Section 2.6 considers rigid rotation, a motion with no degrees of freedom, drawing another parallel with Bell's paper on relativity: an idealistically fragile disk must shatter if undergoing rigid rotation.

Section 2.7 identifies a rigid motion in Schwarzschild spacetime. A rod lying radially with respect to the gravitational source in such a way that its particles can be labelled by constant values of the Schwarzschild radial coordinate is undergoing rigid motion according to this definition. This brings the discussion back to the problem of relating coordinates to physical measurement, particularly in general relativity theory, and introduces the question of whether it is possible to move a rod rigidly from one radial position to another in the Schwarzschild spacetime. It is suggested that the rod must always occupy the same proper length in the Schwarzschild spatial hypersurfaces in order to achieve this, making the idea of rigid motion seem rather natural.

On the other hand, it is not obviously based on any microscopic physical theory. The parallel with Bell's atom is raised to suggest a way of linking this kind of rigid motion with real physics. Such an atom is briefly discussed in the context of general relativity theory. A version of the equivalence principle is invoked to explain why one might expect an atom to have the same proper diameter when stationary relative to Schwarzschild space coordinates in Schwarzschild spacetime as it would in a flat spacetime.

It is suggested that it is the equivalence principle in some form that allows us to make any physical interpretation of coordinates in general relativity theory. The idea of a Bell rod, made from a string of Bell atoms, is then related to the above notion of a rod undergoing rigid motion, and it is suggested that one should represent a good approximation to the other.