About "Minkowski's Influence on Whitehead" **Ronny Desmet**
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Alfred North Whitehead (1861-1947) is known by many for his *Principia Mathematica* collaboration with Bertrand Russell, and by some for his later philosophical adventures. In order to discover the influence of Hermann Minkowski on Whitehead, however, we must not primarily focus on the mathematics of his Cambridge period (1880-1910), nor on the metaphysics of his Harvard period (1924-1947), but on his involvement with relativity during the London period of his professional career (1910-1924). This involvement culminated in an alternative rendering of Einstein's general theory of relativity in his 1922 book, *The Principle of Relativity with applications to Physical Science.*

A quick look at Whitehead's alternative theory of gravitation is sufficient to characterize it as "Minkowskian." Whereas Einstein's law of gravitation is expressed in terms of a fundamental tensor embodying both space-time geometry and gravitational physics, Whitehead maintains the old division between physics and geometry. His law of gravitation is expressed in terms of two tensors, a gravitational field (his potential mass impetus tensor) against the background of a Minkowskian space-time metric (his Galilean tensor). Whitehead's 1922 bi-tensorial (or, if you like, bimetric) law brings the Poincaré-Minkowksi search for a Lorentz covariant law of gravitation to a close while being to a large extent empirically equivalent to Einstein's 1916 law – the fact that Whitehead's equation and Einstein's share the Schwarzschild solution (as shown by Arthur Eddington in 1924) and the Kerr solution (as shown by Robert Russell and Christoff Wasserman in 1986) accounts for the substantial empirical equivalence.

So, Whitehead's theory is a background-dependent theory of gravitation which, contrary to similar attempts (e.g. the 1911 attempt of Willem de Sitter), did not contradict the observed precession of the perihelion of Mercury. But that is not the only reason for calling Whitehead's theory "Minkowskian." The aim of this paper is to show that, and how, Whitehead arrived at his theory thanks to the influence of Minkowski, both direct and indirect. Even though Whitehead's writings are characterised by an absence of bibliographical data and by insufficient footnote-references to his sources, based on Victor Lowe's 1985 & 1990 Whitehead-biography and on the information Whitehead gives in the prefaces to his books, it is possible to point at the Minkowskian sources of the Whiteheadian alternative to Einsteinian relativity, and to give a mathematical reconstruction of how Whitehead arrived at his alternative.

When Whitehead, in the aftermath of the famous British expedition testing Einstein's general theory of relativity in the context of the 1919 solar eclipse, was asked to write an article on "Einstein's theory" for *The Times Educational Supplement,* he included the following tribute to Minkowski: "Before passing on to Einstein's later work a tribute should be paid to the genius of Minkowski. It was he who stated in its full generality the conception of a four-dimensional world embracing space and time, in which the ultimate elements, or points, are the infinitesimal occurrences in the life of each particle. He built on Einstein's foundations and his work forms an essential factor in the evolution of relativistic theory." (February 12, 1920) The aim of this paper is to show that the latter sentence is not only true in general, but also holds for Whitehead in particular – Minkowski's work forms an essential factor in the development of Whitehead's relativistic theory.

In order to reach this aim, in a first movement, based on a number of biographical details and on Whitehead's scarce references, my paper identifies as the most relevant Minkowskian sources of Whitehead's theory of gravitation: Minkowski's 1908 Cologne lecture, "Space and Time"; E.B. Wilson and G.N. Lewis's 1912 memoir, "The Space-Time Manifold of Relativity"; Ebenezer Cunningham's 1914 book, *The Principle of Relativity;* Ludwik Silberstein's 1914 book, *The Theory of Relativity;* and finally, Silberstein's 1918 article, "General Relativity without the Equivalence Hypothesis." In a second part, the paper will give a mathematical reconstruction of Whitehead's theory of gravity based on elements drawn from the Minkowskian sources identified in the first part, and on elements drawn from Einstein's work (let's not forget that Whitehead's theory is *ad hoc*).

In "Space and Time" Minkowski writes: "The whole universe is seen to resolve itself into similar world-lines, and [...] in my opinion physical laws might find their most perfect expression as reciprocal relations between these world-lines." And after linking the electrodynamic relations between the world-lines of point-charges to the Maxwell-Lorentz electron theory and the Liénard-Wiechert potentials, Minkowski expresses his belief that the gravitational relations between the world-lines of point-masses should be treated just like the electrodynamic relations in the case of point-charges, and hence, that the resolution of the universe in world-lines can be seen as "the true nucleus of an electromagnetic image of the world."

Whitehead, whose ultimate motivation – in all three periods of his career – is best characterized as an urge for unification, has turned Minkowski's belief in the possibility to unify the electrodynamics of point-charges and the gravitation of point-masses in terms of reciprocal relations of world-lines and retarded potentials into a fully fledged theory of causally efficient "historical routes of events" characterised by definite charges and masses – the causal efficacy being described in terms of retarded potentials in Minkowskian space-time. Whitehead succeeded in turning Minkowski's belief into a theory because he stood on the shoulders of Wilson and Lewis, of Cunningham, and of Silberstein, and because he intelligently combined some of the elements in their writings, e.g., Cunningham's Lorentz covariant four-vector expression of the Liénard-Wiechert potentials with Silberstein's disentanglement of Einstein's general relativistic merger of geometry and physics.

Moreover, whereas Minkowski considered his world-postulate as "the true nucleus of an electromagnetic image of the world," Whitehead's theory of relativity can be considered as the nucleus of his later process metaphysics. This last statement, however, goes beyond the content of the paper here proposed. The paper is limited to throwing some light on Minkowski's influence on Whitehead by means of the identification of the scientific sources of Whitehead's theory of gravitation, and by means of its mathematical reconstruction. And while doing just that, it also shows that Whitehead is unjustly neglected in the history of the British reception of relativity (e.g., in Andrew Warwick's 2003 *Masters of Theory* and in Thomas Ryckman's 2005 *The Reign of Relativity*), for Whitehead's case is one of the most important illustrations of the well-known fact (stressed, e.g., by Scott Walter) that Minkowski's work played a major role in the reception of Einstein's relativity, especially among German mathematicians (as stressed, e.g., by Leo Corry) and their British colleagues (as stressed, e.g., by José Sanchez-Ron).