QUANTUM REAL NUMBER MINKOWSKI SPACETIME

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I. INTRODUCTION

At the scales of spatial volumes and times associated with micro-physical entities, the structure of space-time is different from that used in classical mechanics. In the Gauss - Riemann approach to geometry the location of points in the classical space-time is described using coordinates which are multiplets of standard real numbers. They are unsuitable for the microscopic world because their physical interpretation as spatial and temporal coordinates presupposes the existence in principle of rigid rods and ideal clocks[6]. Some authors, see Toller [1] and its references, have described the space-time of micro-physics using coordinates which are multiplets of operators that belong to a non-commutative algebra. For example, if the micro-system is described using a unitary representation of the Poincaré group $\mathcal{P}$ then the coordinates are given by operators $\hat{X}^{\mu}$ belonging to the infinitesimal representation of the enveloping algebra of $\mathcal{P}$. Then a connection with the classical outcomes of measurements is obtained through the eigenvalue-eigenvector link which runs into difficulties for operators, like $\hat{X}^{\mu}$, that do not have point spectrum[4]. In the program that we follow, the operators are taken to label physical qualities and their quantitative values are given by quantum real numbers (qrumbers)[9, 13]. Each massive particle may consider itself to be a quantum real number reference frame in much the same way as classical frames of reference are set up because that all "measured" values are given by qrumbers that are not disturbed by uncertainty relations. On the other hand, a laboratory measurement is a process in which the quantum real number yields a standard real number to a given precision[13], here the uncertainty relation arises as a limitation on the product of the precisions of the position and its conjugate momentum.

In the standard quantum formulation the physical system is assumed to be in a pure state described by a vector ψ in a Hilbert space $\mathcal{H}$. In the quantum real numbers (qrumbers) approach the system is assumed to be in a quantum condition W defined as an open subset of the state space[15], $\mathcal{E}_{\mathcal{S}}(\mathcal{M})$, which is contained in the convex hull of projection operators $\hat{P} = |\phi\rangle\langle\phi|$ onto one-dimensional subspaces spanned by unit vectors $\phi \in \mathcal{D}^{\infty}(U) \subset \mathcal{H}$. [16]

An event in Minkowski space-time is a point in a four-dimensional manifold. Events correspond to elementary physical phenomena which can be given temporal and spatial coordinates. This correspondence can be maintained in the microscopic world when these classical variables are re-expressed in terms of qrumbers. The problem of the existence of a "background" reference frame[7] is the same as in classical mechanics except the logic is intuitionistic and the qrumbers may exist to less than full extent[9].

The qrumber coordinates of the quantum Minkowski space-time are given by quadruplets $x_Q^\mu(W),\ \mu = 0, 1, 2, 3$ where the continuous functions x_Q^μ with a common domain W are given by the formulae $x_Q^\mu(\hat\rho) = Tr\hat\rho\hat X^\mu$ for all $\hat\rho \in W$. The qrumbers form a field. [17]

The unitary representation U induces a representation of the proper orthochronous Poincaré group $\mathcal{P}$ in the quantum Minkowski space-time, such that for any element (a^μ, A^μ_ν) in $\mathcal{P}$ and any open set W, the event $x_Q^\mu(W) \to a^\mu 1_Q(W) + A^\mu_\nu x^\nu_Q(W)$. Here $a^\mu \in \mathcal{T}$, the space-time translation group, and $A \in \mathcal{L}$, the proper orthochronous Lorentz group. Therefore the usual Minkowskian tensor calculus is applicable.

Because the vacuum state and single-particles states cannot be used to determine an event,[1, 2] we only use representations with positive mass. We follow Jaekel and Reynaud in using representations of the conformal group[2] to construct the points of space-time and discuss how the topology of the qrumber Minkowski space-time can explain some "non-local" aspects of the experiments for the double slit and Bell's inequality [12].

The standard quantum mechanical expectation values are related to qrumber values because the expectation value $Tr\hat\rho\hat A$ of the quality $\hat A$ in the state $\hat\rho \in V$ defines an order theoretic infinitesimal qrumber. [18]

[1] M. Toller. *Events in a Non-Commutative Space-Time.* Phys.Rev.**D70** (2004) 024006. physics.hep-th/0305121

[2] M-T. Jaekel and S.Reynaud, *Conformal symmetry and quantum relativity.* Found. of Phys. **28**, 439 (1998)

[3] B. Riemann, *On the hypotheses which lie at the Foundations of Geometry*, in M. Spivak, *A comprehensive introduction to Differential Geometry*, **II**, Publish or Perish, Boston (1970).

[4] J. V. von Neumann: English translation: *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, Princeton, 1955. §VI.3 pp 442-445.

[5] Inoue A, *Tomita-Takesaki Theory in Algebras of Unbounded Operators*, Lecture Notes in Mathematics; 1699, Springer (1999), K. Schmudgen, *Unbounded Operator Algebras and Representation Theory*, Operator Theory: Vol. 37, Birkhauser Verlag (1990).

[6] A. Einstein, *Physics and Reality*, Journal of the Franklin Institute, **221**, 3, March 1936. Republished in *Ideas and Opinions*, pp 296-299 (Crown Publishers, New York, 1954).

[7] M. Dickson, *Quantum Reference Frames in the Context of EPR*, Phil. of Science, **71**, 655 (2004).

[8] L. N. Stout,*Topological Properties of the Real Numbers Object in a Topos.* Cahiers Top. et Geom. Diff. **XVII**, 295 (1976).

[9] M.Adelman and J.V. Corbett: *Quantum Mechanics as an Intuitionistic form of Classical Mechanics*, Proc. Centre of Maths and its Applications, pp15-29, ANU, Canberra (2001).

[10] J.V.Corbett *The Pauli Problem,state reconstruction and quantum real numbers*, Reports on Mathematical Physics, **57**,53-68, (2006) .

[11] J. V. Corbett and T. Durt, *Quantum Mechanics interpreted in Quantum Real Numbers* quant-ph/0211180, 1-26, (2002).

[12] J. V. Corbett and T. Durt, *Quantum Mechanics as a Space-Time theory* , quant-ph/051220, 1-27, (2005).

[13] J.V.Corbett and T Durt, *Collimation processes in Quantum Mechanics interpreted in Quantum Real Numbers*, (2007) preprint.

[14] J. V. Corbett, *Mathematical Structure of Quantum Real Numbers*, in preparation, (2007).

[15] The O^*-algebra $\mathcal{M}$ is generated by the infinitesimal representation of the symmetry group on $\mathcal{D}^\infty(U)$, the set of C^∞-vectors for U[5]. The states are the strongly positive linear functionals on $\mathcal{M}$, normalised to 1 on its unit element $\hat{I}$. The state space $\mathcal{E}_\mathcal{S}(\mathcal{M})$ is the set of all states.

[16] Given an operator $\hat{A} \in \mathcal{M}$, a standard real $\epsilon > 0$ and $\hat{\rho}_0 \in \mathcal{E}_\mathcal{S}(\mathcal{M})$, the sets $\mathcal{N}(\hat{\rho}_0; \hat{A}; \epsilon) = \{\hat{\rho}\ ; |Tr\hat{\rho}\hat{A} - Tr\hat{\rho}_0\hat{A}| < \epsilon\}$ form an open sub-base for the topology on $\mathcal{E}_\mathcal{S}(\mathcal{M})$; the subsets $\Lambda(\hat{\rho}_0; \delta) = \{\hat{\rho}; Tr|\hat{\rho} - \hat{\rho}_0| < \delta\}$ for fixed $\hat{\rho}_0$ and $\delta > 0$ form an open base[14].

[17] The field of qrumbers in the topos Shv($\mathcal{E}_\mathcal{S}(\mathcal{M})$) of sheaves on $\mathcal{E}_\mathcal{S}(\mathcal{M})$ is generated by functions of the form $a_Q(\rho) = Tr(\rho\hat{A})$ for each operator $\hat{A} \in M$. The qrumbers have sufficiently many properties to develop an integral and differential calculus. Their logic is intuitionistic.

[18] a is an order theoretical infinitesimal qrumber if there is no open set on which $a > 0 \vee a < 0$ is true.