Abstract:

Minkowski's Proper Time and the Clock Hypothesis

In his classic paper of 1908, Minkowski introduced the concept of *proper time*, a concept surrounding which several confusions persist, especially in matters relating to *acceleration*. In this paper I attempt to untangle these confusions and clarify the importance of Minkowski's profound contribution to the ontology of modern physics.

1) Proper time is still often taken to be simply *local time*, the time co-ordinate measured by a clock at rest in a local inertial frame. This is encouraged by an analogy with *proper length*, which is the length of a body in its rest frame, as well as by the numerical equivalence of the value of the proper time elapsed for a body moving along an inertial path with the time elapsed as measured by the time-co-ordinate in its rest frame. As defined by Minkowski, however, proper time is an integral along an arbitrary timelike curve between 2 events, and its value is independent of the inertial co-ordinate system used to calculate it. Unlike proper length, its value is not specific to a particular frame, the rest frame of the body. I explore this failure of the analogy with proper length, and its implications for the distinction between space and time.

2) Relatedly, it is often stated that Special Relativity (SR) applies only to objects in inertial motion, and not to accelerated systems, for which an appeal to General Relativity (GR) is necessary. This is encouraged by a faulty application of the Equivalence Principle (EP), as well as by a misinterpretation of Einstein's 1918 paper on the Twin Paradox (a. k. a. the Clock Paradox). On this misreading it is thought that since the acceleration of the travelling twin is responsible for the difference in the twins' ages (i.e. in the proper times of their journeys), and accelerations fall outside the scope of the special theory, and since (by the EP) this acceleration will be equivalent to a gravitational field resulting in a *gravitational time dilation*, the paradox must receive its explanation in GR. On the contrary, the paradox receives a complete explanation within SR, which is perfectly applicable to accelerated motions, and in his 1918 paper Einstein is concerned to show that this SR explanation is entirely compatible with the equivalent explanation in GR. The moral of the story is that time dilation in SR arises from the different paths that are the results of the different accelerations experienced by the twins, but not from the acceleration itself, a fact which (by the EP) has an analogue in gravitational time dilation: this is

due to the difference in gravitational potential at 2 points in the field, rather than being caused by the action of the field itself.

3) A much more subtle set of issues surrounds the "clock hypothesis", according to which "acceleration as such has no effect on the rate of the clock, i.e. its instantaneous rate depends only on its instantaneous speed" (Rindler, Essential Rel. 43). The first issue is that on the above analysis it seems otiose to make the clock hypothesis a separate assumption in Special Relativity, where proper time's being an integral of the elements $d\tau$ along a timelike curve follows mathematically from the Minkowski geometry, itself a direct consequence of the Lorentz invariance of all the laws governing non-gravitational interactions. But several authors claim that the clock hypothesis is a separate physical assumption, dependent on the dynamics of real clocks, and necessary for the physical application of Minkowski's mathematics to real physical systems. In particular, Harvey Brown and Oliver Pooley cite the example of Weyl's failed attempt at a unified theory of electromagnetism and gravitation, in which (as Einstein pointed out in criticizing the theory) the proper time elapsed would not only vary with the path through spacetime, but would in addition be affected by electromagnetic fields traversed. They claim that the very possibility of such a second time dilation effect of dynamical origin shows that Einstein has made two assumptions relevant to proper time: (i) that locally the metric is Minkowskian, and (ii) that the consecutive elements $d\tau$ in consecutive LIFs are all that contribute to total proper time elapsed, i.e. that there is no contribution over and above that of the instantaneous velocities in timelike paths that are not geodesics. I examine this and related arguments given by Wolfgang Rindler that the clock hypothesis is a separate physical assumption needed for the application of relativity to dynamics, and conclude that its separate assumption is no more necessary in relativity theory than in Newton's physics. It is not an independent hypothesis, but simply a description of the behaviour of an ideal clock as predicted by (classical, special and general) relativity theory. Its failure would entail the falsity of relativity theory in the form proposed by Einstein, as Weyl had sought to demonstrate. Moreover, understood as a criterion for an ideal clock, the clock hypothesis has in turn been conflated with the separate problem of whether the restorative acceleration of the mechanism within any real system acting as a clock is sufficiently great (relative to the acceleration undergone by the system) that the system will be able to approximate such an ideal clock.