A space-time formalism with negative mass

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Abstract

Space-time can also be visualized by using the three spatial dimensions and proper time (times c) as a fourth dimension. The time t is then equal to the length of a worldline in these four dimensions (divided by c).

$$c^2dt^2 = c^2d\tau^2 + dx^2 + dy^2 + dz^2 \tag{1}$$

The length of a path of a particle in this flat space-propertime is always positive, and because the length of the path increases, so does time t. Particles moving downwards in such a diagram (with decreasing τ along the path) are identified as anti-particles. So anti-particles move forward in time t, but backwards in proper time. Stueckelberg and Feynman have made similar suggestions.

A similar visualization is also possible for other four-vectors such as the energy-momentum four-vector. Such a vector could then be visualized by using the three momentum-dimensions (times c) and the mass (times c^2) as a fourth dimension. Note that this allows us to have negative mass. The length of a vector in these four dimensions equals energy, which is always positive.

Just as with negative proper-time, we will identify particles with negative mass as anti-particles. Negative mass can cause problems and inconsistencies when used with some formulas for energy and momentum. Therefor a more careful definition of energy-momentum is used, that includes the proper time:

$$p^\mu = m\frac{dx^\mu}{d\tau} = (\gamma\,|m|\,c, \gamma\,|m|\,\vec{v}) \tag{2}$$

Because antiparticles have negative mass and move backwards in proper time in this model, the minus-signs of the mass m and $d\tau$ will cancel. So also no negative energy occurs and the momentum of the antiparticles is in the direction of their velocity.

We see that for some things the result is independent of the sign of the mass. But in other areas there is a difference. Currently two different Dirac equations are used to describe particles and anti-particles. According to Griffiths [1]:

> Notice that whereas the u's (particles) satisfy the momentum space Dirac equation in the form
>
> $$(\gamma^\mu p_\mu - mc)\,u = 0 \tag{3}$$
>
> the v's (anti-particles) obey the equation with the sign of p_μ reversed:
>
> $$(\gamma^\mu p_\mu + mc)\,v = 0 \tag{4}$$

In our formalism we would say that for an anti-particle the sign of the mass is reversed, but not the sign of p_μ. In our model it is not needed to have different Dirac-equations for particles and anti-particles. By using negative mass, the sign for the mass in equation 3 is automatically reversed for anti-particles. According to equation 2 the sign of p_μ will remain the same.

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Another area is gravitation. Let us analyze the following stress-energy tensor.

$$T^{\mu\nu} = m \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} \tag{5}$$

For an anti-particle m is negative and so is $d\tau$. But because $d\tau$ appears twice, it doesn't cancel the minus-sign of the mass now. In this model it is predicted that antimatter has anti-gravitational properties, and could be the dark energy in the universe. Experiments are proposed to test this.

References

[1] D. Griffiths, Introduction to elementary particles (John Wiley & Sons, New York, 1987).

[2] H. Bondi, Negative Mass in General Relativity (Rev. Mod. Phys. 29, 423-428, 1957)

[3] John P. Costella, Bruce H. J. McKellar, Andrew A. Rawlinson; Classical antiparticles (American Journal of Physics, September 1997, Volume 65, Issue 9, pp. 835-841)