# Is spacetime a field?

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It is often claimed that the core of the Theory of General Relativity (GR) is that it shows that the gravitational field is just an aspect of spacetime itself. Indeed, Einstein's idea of *associating* the gravitational field with the geometry of spacetime was one of the most beautiful ideas in the history of physics: the same mathematical object — the metric tensor field — represents both gravity and the geometry of spacetime. However, it remains controversial in what sense gravity and geometry are associated with each other. My aim in this paper is to clarify the various possibilities about how this could be understood.

The role of the metric as describing the geometry of spacetime is in fact a peculiarity of Riemannian geometry, in which the metric determines the other geometric quantities (i.e. the connection and the curvature tensor) uniquely — in stark contrast to e.g. affine geometry which takes the connection as the fundamental geometric object. In Riemannian geometry, the metric determines lengths and angles, light cone structure and even the possibility of distinguishing between past and future. Thus, the metric can surely be seen as the representative of the geometry of spacetime.

But the role of the metric as describing gravity is less clear. Does it represent the gravitational field or the gravitational potential? If the latter, what is then the representative of the gravitational field? I will argue that the connection is the proper representative of the field, one argument for this being that only then will the principle of equivalence — the very starting point of GR — be still valid in the full theory.

So what exactly does it mean to say that GR associates the geometry of spacetime with the gravitational field? There are essentially three possibilities:

1. Describing both gravity and geometry in terms of the metric tensor means eliminating the picture of gravity as a force (field): it is rather a manifestation of spacetime geometry. Thus, *GR is the geometrization of gravity.*

2. The possibility of associating both geometry and gravity with the metric tensor shows that spacetime geometry is nothing more than the gravitational field itself. It is not gravity that is a manifestation of the geometry of spacetime. Rather, the geometry of spacetime is a manifestation of the gravitational field. Thus, *GR is the fieldization of geometry.*

3. We should take the fact that *both* gravity and geometry are described by the metric tensor to heart: neither the geometry-perspective (1.) nor the field-perspective (2.) are privileged. Rather, *GR can be seen as asserting the gravitational field and the geometry of spacetime to be indistinguishable.*

The first position is taken by the majority of textbooks on GR [1], the second by many physicists working on quantum gravity approaches [2] and also by some philosophers of physics [3]. The two positions may be regarded as *two different forms of realism* towards GR — so GR probably cannot be cast along the familiar paths of the general realism/anti-realism debate.

The third position may be called *egalitarianism.* But there is something deeper about it: it is this position which seems to take the principle of equivalence most seriously.

To sum up: I will investigate whether GR is a geometrization of the gravitational field or a fieldization of the geometry of spacetime. I will argue for the third position above: GR seems to be both, and quite a good deal of its potential derives from the possibility to switch between field-picture and geometry-picture.

I have yet just spoken about the *geometry* of spacetime, but avoided talk about "spacetime itself". Indeed, one may well ask: is there more to spacetime than "its" geometry? It seems that all the paradigm properties of spacetime are described by the metric field. I will thus finish by discussing whether there are good reasons for regarding the metric field as describing spacetime *completely*, i.e. for regarding the manifold of spacetime points as a mathematical artifact, not corresponding to something real. Finally, I will point out what consequences such a stance would have for the above three ways of interpreting GR.

## References

[1] C .W. Misner, Kip. S. Thorne, and John A. Wheeler. *Gravitation.* Freeman, 1973.

[2] Carlo Rovelli. Halfway through the woods: contemporary research on space and time. In John Earman and John Norton, editors, *The Cosmos of Science*, pages 180–223. University of Pittsburgh Press, Pittsburgh, 1997.

[3] Harvey R. Brown. *Physical Relativity. Space-time structure from a dynamical perspective.* Oxford University Press, 2005.