Singular structure and the nature of space-time

Vincent Lam
University of Lausanne, Switzerland

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The standard mathematical representation of space-time within contemporary physics in terms of a set of points endowed with certain topological and smooth differential properties plays an important, but actually ambiguous, role in the recent philosophical debate about the nature and the ontological status of space-time. On the one hand, the manifold model of space-time seems to constitute a mandatory framework for fundamental physics, and especially for (quantum) field theories and the theory of (classical) general relativity (GR). This characterization of space-time as a set of points with certain properties allows one to focus on local properties and local entities, these latter notions being (in general) closely linked to the concept of a point (and its neighbourhood). But on the other hand, even if such a framework is in fact successful for doing physics, it may lead to some confusion when it comes to the ontological commitment with respect to space-time, at least if some global aspects are not taken into account (and even if a clear distinction between the mathematical model and the physical world is respected). Indeed, following this stance, the discussion about the ontological status of space-time may be considered (naively) to be equivalent (and reducible), or at least to be interrelated, to the one about the ontological status of its parts. In particular, such an alleged link has been (sometimes implicitly) used in order to argue that substantivalism about space-time is somehow committed to a (strong) substantivalist view about space-time points or regions, in the sense of physical entities possessing intrinsic properties. Then, it is only within this (narrow) framework that the principle of active general covariance (or of invariance under active diffeomorphisms) may constitute a (hole-type) argument against substantivalism. There are of course alternative (and more 'sophisticated') substantivalist positions that are not subject to such an argument. However, the point that I want to stress here is that space-time may actually possess some global features that break the above mentioned link; their careful study and interpretation may indeed prevent us from drawing any (misleading) conclusions about the nature and the ontological status of space-time that are uniquely based on local (pointlike) considerations.

In this sense, this paper aims to investigate the possible relevance that space-time singularities, if taken seriously, may bear for the debate about the nature and the ontological status of space-time. 'Seriously' means here that they are not merely considered as an indication for the 'breakdown' of GR [1]. Indeed, they can well be considered as problematic in a certain sense (as possibly implying infinite values for certain physical quantities for instance) but nevertheless as involving some fundamental features of space-time (at least as far as their genericity is insured by the validity of the hypotheses of the singularity theorems). With this I am not only referring to the alleged 'common wisdom' according to which space-time singularities will be avoided if quantum effects (or the possible quantum or discrete nature of space-time) are taken into account (there are indeed some recent interesting results in this sense in Loop Quantum Cosmology).

Rather, already at the classical level, I want to consider what may be the possible meaning for the nature of space-time of some aspects of this singular structure. The notion of a space-time singularity actually covers various different (and sometimes problematic) aspects that cannot all be captured in one single definition. I only want to focus here on the aspects that may have some direct relevance for my concern. In particular, within the framework of the standard account of space-time singularities that relies on the fundamental notions of extension, curve incompleteness, and boundary, I consider the today well-known difficulties to characterize space-time singularities in terms of local properties or in terms of local entities [2]. Indeed, on the one hand, space-time singularities are 'singularities' of the space-time structure itself and therefore cannot be merely described in terms of Lorentz manifold points. On the other hand, because of 'problematic' topological behaviour (like non-Hausdorff behaviour [3] or curve incompleteness occurring in a compact space-time region) space-time singularities represented by points in a boundary attached to the Lorentz manifold cannot be considered in any usual sense as being 'localized' with respect to this latter manifold. With respect to these difficulties, space-time singularities (whose very denomination is indeed misleading) may be best understood as a global feature of space-time, in the sense of a space-time property that cannot be ascribed to any particular space-time point, but rather only to the entire space-time as a whole. Consequently, I argue that the singular behaviour of space-time can be well conceived (together with other aspects of space-time, like some properties of the gravitational energy for instance) as an indication of certain possible global aspects of the space-time structure at the fundamental level. Moreover, I argue that this possible fundamental feature of space-time fits well within the framework of a structural realist interpretation of space-time.

In order to further evaluate the possible implications of the (global) singular feature of space-time, I propose to consider some recent developments in the algebraic approaches of GR with respect to this singular feature. These alternative approaches to space-time exploit the full equivalence of the usual geometric conception of the smooth Lorentz manifold model of space-time with a purely algebraic conception in terms of the algebra of the smooth real functions defined on it [4]. This equivalence allows one to consider this latter algebraic structure as primary and the smooth Lorentz manifold as a secondary, derived notion. Moreover, this algebraic conception of space-time reveals a new perspective on the singular behaviour of space-time and is open to sheaf-theoretic and non-commutative generalizations, which provide uniform (and 'singularity-free') abstract algebraic descriptions of space-time [5]. Space-time singularities become then artifacts of the (commutative) smooth differentiable manifold model commonly used to describe space-time at a less fundamental level. Within the non-commutative algebraic framework, there is actually no notion of locality anymore, at least as far as this notion relies on the concept of space-time point (and its neighbourhood): in general, the concept of manifold point cannot be derived anymore from the now non-commutative algebraic structure, so that the very concept of a space-time point is not represented at this level. Space-time is therefore described at the fundamental level as a pure global structure that is not based on the notion of space-time points ('pointless' space) [6].

Even if the algebraic approaches of GR are still rather speculative, my point is that this emphasis on the global aspects of space-time (which are actually already there within 'standard' GR) constitutes an interesting way to address the 'singularity problem' that may shed some new light on the debate about the ontological status of space-time. The various positions in this debate should indeed provide a satisfactory account of these global aspects or at least of the singular feature of space-time.

References

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