

# A Physicist Viewpoint on Space-time, Simultaneity, and Accelerated Observers.

Luca Lusanna  
*Sezione INFN di Firenze*  
*Polo Scientifico*  
*Via Sansone 1*  
50019 Sesto Fiorentino (FI), Italy  
E-mail LUSANNA@FI.INFN.IT

The Newtonian notions of 3-space and time have been replaced by Minkowski space-time, a pseudo-Riemannian 4-manifold with Lorentz signature, because of the overcoming of the absolute notion of simultaneity. Even if the chrono-geometrical structure of Minkowski's space-time is fixed and non-dynamical, the establishment of a metrology (rods and clocks) is necessarily observer-dependent and requires the choice of a *convention* about the synchronization of distant clocks. Lacking this, the notions of 3-space, one-way velocity of light and spatial distance between space-like-separated events are not definable. Therefore while the world is 4-dimensional, the notion of 3-space is observer-dependent. Any inertial observer with Cartesian 4-coordinates, using light signals and Einstein's convention defines the space-like hyper-planes of constant time as simultaneity 3-spaces. As shown by the lasting debate about the conventionalism of simultaneity, other possibilities are open for both inertial and non-inertial observers. These unorthodox choices lead to different local description of phenomena (the 1+3 point of view). Since actual observers are always accelerated, a big effort has been undertaken to the effect of building associated 4-coordinate systems that generalize Fermi coordinates. Yet, all of them failed in defining a good notion of simultaneity. This fact is exemplarily shown by the lack of consensus about the definition of a satisfactory relativistic rotating 4-coordinate system with a well defined notion of 3-space (see the unending debate about the rotating disk and the Sagnac effect).

A possible way out [1] from these problems is the 3+1 point of view together with Møller's conditions for the admissibility of the notion of simultaneity and thereby of 3-space. Accordingly, in order to describe relativistic system in 3-space, one has to make a 3+1 splitting of Minkowski space-time by means of an arbitrary, admissible, foliation whose leaves are just space-like 3-spaces. Then, each 3-space is both a simultaneity surface (i.e., a convention for the synchronization of clocks) and a Cauchy surface for the equations of motion on which the initial data for the predictable evolution must be given. This approach allows to associate admissible notions of simultaneity to arbitrary accelerated observer.

A particular development has been the formulation [2] of isolated relativistic systems (particles, strings, fields) as "parametrized Minkowski theories". In this approach all the admissible foliations and the associated notions of simultaneity are "gauge equivalent" because of a special type of general covariance. Since the high precision of the new atomic clocks ( $10^{-15} - 10^{-16}$ ) allow to test relativistic effects till the order  $1/c^3$ , this approach turns out to be relevant [1] for space navigation (recall the Global Positioning System).

Since in general relativity the chrono-geometrical structure of space-time becomes dynamical, a new problem arises from the so-called *Hole Argument*. Actually, as shown in

Ref. [3] (see also Pauri's talk at this Conference) there is the need to physically individuate the mathematical points of the space-time 4-manifold. This requires in its turn a correct treatment of the initial value problem of Einstein's equations and an identification of the Dirac observables for the gravitational field. On the other hand, such treatment can be done correctly only in the Hamiltonian formulation, which just requires the 3+1 point of view for defining the notion of Cauchy and simultaneity 3-surface [4]. The new important facts are now that: (i) the admissible notions of simultaneity and the associated family of accelerated observers are *dynamically selected* by the solutions of Einstein's equations, and that: (ii) the general covariance entail the "gauge equivalence" of such notions of simultaneity.

In conclusion, if we wish to have predictability in both special and general relativity, we have to start from an admissible convention of synchronization of distant clocks as a prerequisite for both the initial value problem and for metrology. Since in these theories the descriptions of phenomena given by different accelerated observers with different associated conventions are "gauge equivalent", it turns out that there is no absolute notion of "relativistic 3-space" but only "observer-dependent 3-spaces" connected by gauge transformations living in the 4-dimensional space-time.

## REFERENCES

- [1] D.Alba and L.Lusanna, *Simultaneity, Radar 4-Coordinates and the 3+1 Point of View about Accelerated Observers in Special Relativity* (gr-qc/0311058).
- [2] L.Lusanna, *The N- and 1-Time Classical Descriptions of N-Body Relativistic Kinematics and the Electro-magnetic Interaction*, Int.J.Mod.Phys. **A12**, 645 (1997).  
L.Lusanna, *Towards a Unified Description of the Four Interactions in Terms of Dirac-Bergmann Observables*, invited contribution to the book *Quantum Field Theory: a 20th Century Profile* of the Indian National Science Academy, ed. A.N.Mitra (Hindustan Book Agency, New Delhi, 2000) (hep-th/9907081).
- [3] L.Lusanna and M.Pauri, *General Covariance and the Objectivity of Space-Time Point-Events: The Physical Role of Gravitational and Gauge Degrees of Freedom in General Relativity* (gr-qc/0301040).
- [4] L.Lusanna, *The Rest-Frame Instant Form of Metric Gravity*, Gen.Rel.Grav. **33**, 1579 (2001) (gr-qc/0101048).  
L.Lusanna and S.Russo, *A New Parametrization for Tetrad Gravity*, Gen.Rel.Grav. **34**, 189 (2002)(gr-qc/0102074).  
R.De Pietri, L.Lusanna, L.Martucci and S.Russo, *Dirac's Observables for the Rest-Frame Instant Form of Tetrad Gravity in a Completely Fixed 3-Orthogonal Gauge*, Gen.Rel.Grav. **34**, 877 (2002) (gr-qc/0105084).  
J.Agresti, R.DePietri, L.Lusanna and L.Martucci, *Hamiltonian Linearization of the Rest-Frame Instant Form of Tetrad Gravity in a Completely Fixed 3-Orthogonal Gauge: a Radiation Gauge for Background-Independent Gravitational Waves in a Post-Minkowskian Einstein Space-Time* (gr-qc/0302084), to appear in Gen.Rel.Grav.