The Regularity Account of Relative Space

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1/19/04

Debate over whether space is absolute or relative almost invariably assumes a geometry and topology for space – almost universally that it is geometrically and topologically Euclidean. This claim is unproblematic for a substantivalist – it's the spatial manifold itself that has the geometry – but what is a relationist supposed to make of it, given that there are many, many possibilities for geometry and topology? Surely silence is not an option, since geometry makes a difference (in the laws of mechanics, say) in what happens to objects as they move through space.

Certainly, the spatial relations between bodies in a finite collection are embeddable in many geometries, many topologies, and even in spaces with and without some specific topology (Nerlich, 1994, emphasizes orientability). Hence, the embeddability of such a system of relations – a '*configuration*' – does not determine the geometry and topology of space. A popular relationist response is to 'go modal', and add facts about what relations are possible to those about which are actually possessed. Leibniz apparently advocated such a view: '[space] does not depend upon such or such an arrangement of bodies; but is that order, which renders bodies capable of being situated…' (*Leibniz's Fourth letter to Clarke*, §41). More recently, several philosophers have advocated the sophisticated form of modal relationism developed by Manders (1982). The idea of this approach is that if enough is said about what relations can and cannot be instantiated by bodies, then there will only be one topology and geometry in which all possible configurations are embeddable – and that mathematical space will express the geometry and topology of relative space.

There are however a number of problems with modal relationism: for instance, helping oneself to modal facts goes against any empiricist motivations one might have for relationism. But worse is the fact that it does not necessarily solve all the problems that face the relationist: in a geometrically inhomogeneous world, what will happen to a body depends not only on the geometry of space but also *where in space a body is located* – whether it is on course for a region of Euclidean or non-Euclidean geometry say. It is surely untenable for the relationist to admit different possibilities of location without explaining what they mean – such 'primitive permanent possibilities' of location are arguably nothing but (substantial) places in another guise. (Manders' theory does fix the location in bodies in this way, and hence according to him, the whole universe could have been located elsewhere with its system of relations intact. Leibniz of course found such '*Leibniz shifts*' absurd; certainly it is hard to see how it could be true in a relational theory!)

I propose an analysis of possible relations and location in terms of relationally acceptable properties. Briefly, all there is to being the geometry and topology of relative space is being the geometry and topology of the *simplest* mathematical space in which the entire history – from start to finish – of relations between bodies is embeddable. That is, talk of geometry and topology is in fact merely a device for simplifying talk about the regular ways in which bodies are related to each other. (Similarly, according to the Mill-Ramsey-Lewis regularity account of laws, laws are nothing more than the consequences of the simplest and strongest axiomatization of the entire history of events.) According to the regularity account of relative space, a set of relations are possible if they are embeddable in the space determined by the history of relations, and the 'position' of a body is understood in terms of the embedding of the history of relations – the regularity account sees both kinds of facts as merely complex facts about the history of relations (and, of course, the relevant notion of simplicity).

In conclusion I consider some consequences of and objections to the theory: for instance, what if the criterion of simplicity fails to fix a unique geometry? (Then some spatial properties are indeterminate.) How does the theory relate to a plausible mechanics or physics of curved spacetime? (Given the affinity to Mill-Ramsey-Lewis, it extends very naturally.) Does the theory allow Leibniz shifts? (No; if a space has the symmetries to allow such shifts, then it is indeterminate where the universe is located.)

H.G. Alexander (ed), *The Leibniz-Clarke Correspondence*, Manchester University Press, 1956.

K.L. Manders, "On the Space-Time Ontology of Physical Theories", *Philosophy of Science* **49** 1982: 575-590.

G. Nerlich, *The Shape of Space*, Cambridge University Press, 1994.