

SPECIAL RELATIVITY PROHIBITS SPACELIKE CAUSATION

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We refer to the rather inconclusive discussions for and against the claim that the theory of special relativity $\text{Th}(\text{SR})$ “does not prohibit spacelike (faster-than-light) causation”. Despite an extensive search we have not found in the literature (not even in Weyl’s monumental work) a formal demonstration either for or against this claim.

A first such formal demonstration *against* this claim is given here. Adding to $\text{Th}(\text{SR})$ the assumption A of “a causal connection between spacelike events” leads to a contradiction. Since $\text{Th}(\text{SR}) \wedge A$ is inconsistent, whence, $\text{Th}(\text{SR}) \vdash \neg A$, and $\neg A$ is a *theorem* of $\text{Th}(\text{SR})$. The formal demonstration of the theorem $\text{Th}(\text{SR}) \vdash \neg A$ assumes no topology (adding topology strengthens the result) and no dynamics, it is frame independent and coordinate-free.

Consequently, $\text{Th}(\text{SR})$ demonstrably asserts that there is no causal connection between spacelike events. Spacelike events are causally disconnected with respect to the apex of the light-cone assigned to every event in Minkowski space-time. In other words, $\text{Th}(\text{SR})$ prohibits spacelike causation and therefore prohibits spacelike signalling (as causation is a weaker notion than signalling).

Since spacelike causation is *ruled out* by the theorem $\text{Th}(\text{SR}) \vdash \neg A$, if an instance of spacelike causation were shown to exist, even if it could not be used for spacelike (faster-than-light) signalling, then the existence of an instance of spacelike causation would refute $\text{Th}(\text{SR})$ which asserts the truth of $\neg A$.

It is claimed that quantum theory apparently exhibits at the level of individual events instances of spacelike causation (which do not amount to spacelike signalling according to the claim) in the shape of causal connections - referred to as “nonlocal interactions” - between pairs of spacelike events.

In the presence of the theorem $\text{Th}(\text{SR}) \vdash \neg A$, if such instances of spacelike causation (“nonlocal interactions”) were shown to exist, or were even presumed to exist, then quantum theory and special relativity would be *incompatible*. The theorem $\text{Th}(\text{SR}) \vdash \neg A$ leads to an *impasse*. And the question it raises is: Can this impasse be resolved ?